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EFFECT OF GRAIN DENSITY ON PLANE BED FRICTION

Václav Matoušek, Vojtěch Bareš, Jan Krupička,
Tomáš Picek, Štěpán Zrostlík

Czech Technical University in Prague, Faculty of Civil Engineering, Czech Republic, v.matousek@fsv.cvut.cz

Sediment-transport experiments with different fractions of model sediment in a tilting flume are analyzed with the aim of evaluating the influence of the bed grain density on bed friction in the lower- and upper plane bed regimes. The water-flow based experiments covered 4 fractions of sediment from which 2 were glass beads and the other 2 plastic grains of different density. The experiments were carried out for a broad range of installed flow rates and flow depths producing a broad range of values of the bed Shields parameter and the delivered concentration of sediment.

The paper discusses and quantifies how different densities of sediment grains affect the bed friction coefficient in flows with no delivered concentration of sediment (the lower plane bed regime), and with high concentration of bed load (the upper plane bed regime).

In the lower plane bed regime, the bed friction coefficient can be determined using the logarithmic law for the hydraulically rough boundary with the equivalent roughness equal to a multiple of the bed grain size. Hence, the roughness of the bed composed of immobile grains is independent of the grain density. In the upper plane bed regime, the situation is more complex.

The bed roughness for the log law can be expressed again as a multiple of the grain size provided that the delivered concentration of the sediment is relatively small. The constant is higher in the upper plane bed regime than in the lower regime and it seems to be affected by the density of mobile grains. If the delivered concentration is high, the log law is no longer appropriate as the distribution of velocity in the flow is no longer logarithmic. Instead, a different friction formula needs to be used which takes also the effect of the grain density into account.

KEY WORDS: lightweight sediment, bed roughness, bed load, tilting flume experiment.

1. INTRODUCTION

In open-channel flows, a relationship between the flow rate and the flow depth is affected by friction at the channel bottom. If the bottom is a granular bed, its resistance depends on the shape of the top of the bed. There are two regimes in which the bed is plane (bed-form free). The lower plane bed (LPB) regime is associated with low bed shear – typically with values of the bed Shields parameter lower than or equal to the critical value for incipient motion of grains at the top of the bed. The upper plane bed (UPB) regime is associated with high bed shear at which the applied shear stress is high enough to wash out bed forms and produce a sheared transport layer at the top of the bed through which bed load is transported.

In the LPB-regime, the top of the plane bed is considered as a hydraulically rough boundary and its friction is described by the logarithmic law of the wall. In the log law, the Nikuradse's equivalent roughness k_s is related to the bed grain size (typically median diameter d_{50}) and the ratio k_s/d_{50} is approximately constant (see e.g. Garcia et al. 2008 for different experimentally determined values of the constant).

In the UPB-regime, the transport layer with bed load affects the bed resistance (Wilson 1987). A typical bed-friction concept considers the top of the upper plane bed as a hydraulically rough boundary with the logarithmic law of the wall even though the top of the bed is eroded. Concepts differ in expressions for k_s/d_{50} relating it either simply to the bed Shields parameter θ (e.g. Wilson 1987, Sumer et al. 1996, Pugh and Wilson 1999, Wilson 2005, Matoušek 2005) or to a larger number of dimensionless parameters (e.g. Camenen et al. 2006, Matoušek and Krupička 2009, Krupička and Matoušek 2010). A more detailed survey of the concepts can be found in Miedema and Matoušek (2014). Experimental data suitable to validate the concepts for open channel flows has remained scarce, particularly in the UPB-regime.

An experimental campaign is in progress in our recirculating tilting flume in Water Engineering Laboratory of Czech Technical University in Prague. It produces experimental data suitable to evaluate friction of both lower- and upper plane beds in open-channel flow. Our analysis based on the flume observations of the UPB-regime suggests that the UBP-regime needs to be split into two sub-regimes. In the lower UPB sub-regime associated with relatively weak transport of sediment, the ratio k_s/d_{50} is virtually independent of the Shields parameter, while k_s/d_{50} is sensitive to θ in the upper UPB sub-regime with intense transport of sediment (Matoušek et al. 2014). In the upper sub-regime, however, expressing the bed friction using the Nikuradse's roughness k_s is questionable as, according to our experiments, the part of the flow depth over which the vertical distribution of longitudinal velocity is logarithmic is narrow and tends to virtually disappear at very high θ (Matoušek et al. 2015).

In our experiments, lightweight sediments are used along with glass-bead fractions in order to reach intense transport of bed load and to enable acoustic measurement of local velocities in the transport layer above the bed. In this paper, we use our experimental results to evaluate an effect of the sediment density on friction of plane beds.

2. EXPERIMENTAL WORK

2.1. MATERIALS

Table 1

Measured properties of tested sediment fractions

Sediment fraction	d_{18} [mm]	d_{50} [mm]	d_{84} [mm]	S_s [-]	w_t [m/s]
HSF30	3.05	3.18	3.25	1.36	0.131
TLT25	3.80	4.15	4.45	1.38	0.106
TK30	3.00	3.00	3.00	2.50	0.309
TK1216		1.49		2.48	0.207

(S_s = relative density of sediment grain, w_t = terminal settling velocity of grain)

Four fractions of model sediments were tested (Table 1). Two fractions were lightweight plastic pellets (HSF30 and TLT25) and the other two fractions were glass beads (TK30 and TK1216). The density of glass beads was almost twice the density of the plastic pellets. The plastic fractions differed from each other primarily by the grain shape. The glass fractions differed in the grain size. The mono-disperse fraction TK30 had a size similar to the plastic fractions and the size of glass beads of the other fraction (TK1216) was one half of the TK30-size.

2.2. EXPERIMENTAL SET-UP AND MEASURED QUANTITIES

The recirculating system is composed of a rectangular flume and connecting pipes. The flume is 0.2-m wide and 8-m long. The pressurized connecting pipes contain the vertical U-tube which serves to determine the mean delivered concentration of grains, C_{vd}, in flowing mixture. The flow rate of mixture, Q_m, is measured using the magnetic flow meter in the upgoing leg of the U-tube. In the flume itself, the water level is measured using ultrasonic probes at several locations along the length of the flume. The position of the top of the bed and the position of the top of the transport layer (layer occupied by transported grains) are observed visually through the glass side walls of the flume at the same locations as the water level. Furthermore, the slope of the flume is measured. These measurements enable to determine the flow depth, H, the thickness of the transport layer, H_{sh}, and the inclination angle of both the bed, ω, and the water surface.

Besides the integral quantities of the flow, the vertical distribution of the longitudinal velocity is measured across the discharge cross section at one location in the flume using three independent methods (Prandtl tube and two acoustic Doppler methods). The system and the measuring techniques are described elsewhere (Matoušek et al. 2015).

2.3. EXPERIMENTAL ESTIMATION OF BED FRICTION PARAMETERS

An estimation of parameters quantifying bed friction is based on the integral quantities measured in steady uniform flow in the flume. The friction velocity at the top of the bed

$$u_{*b} = \sqrt{gR_b sin\omega} \quad (1)$$

in which g = acceleration of gravity and R_b = hydraulic radius of the discharge area associated with the top of the bed, i.e. the flow depth after implementing the side-wall correction.

The bed friction coefficient

$$\lambda_b = 8\left(\frac{u_{*b}HB}{Q_m}\right)^2 \quad (2)$$

in which B = flume width.

The bed friction coefficient is used to estimate the Nikuradse's equivalent roughness of the bed, k_s, in the logarithmic law of the hydraulically rough boundary

$$\sqrt{\frac{8}{\lambda_b}} = \frac{1}{\kappa} ln\left(\frac{B_\lambda R_b}{k_s}\right) \quad (3)$$

in which κ = von Kármán constant (κ = 0.4) and B_λ = log-law constant (B_λ = 11.1 for open channel flow).

3. DISCUSSION OF FRICTION AT TOP OF PLANE BED

3.1. LOWER PLANE BED

In the lower plane bed regime, 5 experimental runs were carried out for two fractions of similar size and shape of grains and very different density (2 runs for TLT25 and 3 runs for TK30). Measured velocity profiles confirmed an existence of the logarithmic profile of velocity across the flow depth for those test runs and hence the use of Equation (3) was justified. The results of the experimental runs showed that $k_s \approx 2 \cdot d_{50}$ was appropriate for both fractions (Table 2). This value of the constant is in agreement with the earlier experimental finding by Kamphuis (1974) for the sand roughness of fixed plane beds in open channels. Hence, the tests confirmed that, for a bed with immobile grains at its top, the rough-boundary log law (Equation 3) is appropriate and the bed roughness is related to the size of the immobile grains through a constant which does not change with the density of the grains.

Table 2

Tests in lower plane bed regime

Run No.	Sediment fraction	$\Delta = k_s/d_{50}$ [-]	θ [-]
1	TLT25	2.2	0.052
2	TLT25	2.2	0.059
3	TK30	0.9	0.043
4	TK30	1.4	0.045
5	TK30	2.1	0.029

3.2. UPPER PLANE BED

A large number of test runs was collected in the upper plane bed regime. The test runs covered both sub-regimes: the one in which the logarithmic velocity profile spans over a considerable part of the flow depth (the lower sub-regime, l-UPB) and the other in which the flow layer with the log profile is narrow or virtually negligible (the upper sub-regime, u-UPB).

In Figure 1, the results are plotted in the abscissa corresponding with Equation (3) and $k_s = \Delta \cdot d_{50}$. ($\Delta$ = constant). The blank points are experimental results for the lower sub-regime (the log-profile important) and the black points are experimental results for the upper sub-regime in which the log-profile is not important. In each plot, the solid line is a result of Equation 3 for a particular value of Δ and the dashed lines are results of the same equation for Δ of ± 50% deviation.

The experimental results for the l-UPB agree reasonably well with the trend given by Equation (3) and a good match can be found provided that an appropriate value of Δ is used (Δ = 7.0 for TK30, Δ = 5.0 for TK1216, Δ = 4.0 for TLT25, Δ = 4.0 for HSF30 in Figure 1). In the l-UPB regime, the values of Δ are higher than in the LPB-regime (Δ ≈ 2 for the LPB-regime) and seem to be sensitive to the grain density (Δ is smaller for the lightweight sediments). This indicates that the use of model lightweight sediment in laboratory experiments may underestimate the real bed roughness of natural sediment at a prototype condition in the l-UPB regime if the sediment density is not taken into account in model scaling.

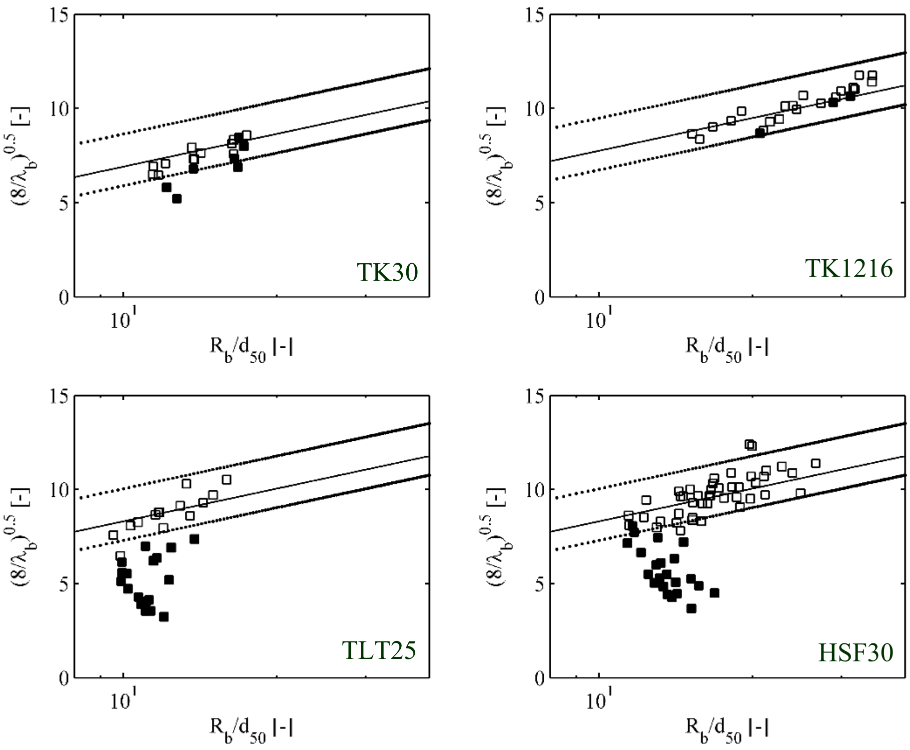

Figure 1 Bed friction coefficient versus relative submergence in upper plane bed regime. Legend: blank squares –measurements in lower sub-regime (l-UPB); black squares – measurements in upper sub-regime (u-UPB); solid line – logarithmic law (Equation 3) for appropriate k_s/d_{50} (Δ = 7.0 for TK30, Δ = 5.0 for TK1216, Δ = 4.0 for TLT25, Δ = 4.0 for HSF30); dashed lines – logarithmic law for 50%-deviation of Δ-value.

In the upper sub-regime, the trend in the relationship between $(8/\lambda_b)^{0.5}$ and R_b/d_{50} is opposite to that in the lower sub-regime (in u-UPB, the λ_b-term decreases with the increasing R_b/d_{50}) and Equation (3) does not work. A power-law formula is proposed (Matoušek et al. 2014) for this sub-regime and it contains terms expressing the effect of the grain density on the bed friction coefficient. The formula respects a result of our experimental observations, which says that a value of the bed friction coefficient tends to be smaller for the lightweight material than for the natural material at the same delivered concentration of bed load, C_{vd}, in the u-UPB regime.

The sensitivity of λ_b to C_{vd} is demonstrated for one plastic fraction and one glass fraction of a similar grain size in Figure 2. The plots for the glass fraction TK30 (left panel) and for the plastic fraction HSF30 (right panel) show the experimental results collected in a narrow range of Q_m from 7.0 to 7.6 l/s (and a broad range of bed surface slopes). It is evident that λ_b increases with C_{vd} much faster if the glass fraction is transported instead of the plastic fraction. This means that intense transport of lightweight bed load contributes much less to the bed resistance than the bed load of natural-material density. On the other hand, the lightweight material is much more mobile and can produce much higher values of C_{vd} at a certain Q_m (compare the maximum C_{vd} of 0.07 for TK30 with 0.16 for HSF30 at $Q_m \approx 7.4$ l/s in Figure 2). Therefore, the maximum value of λ_b reached for this Q_m in the flume is finally bigger for the lightweight material than for the glass material and it is associated with a considerably higher flow rate of lightweight sediment.

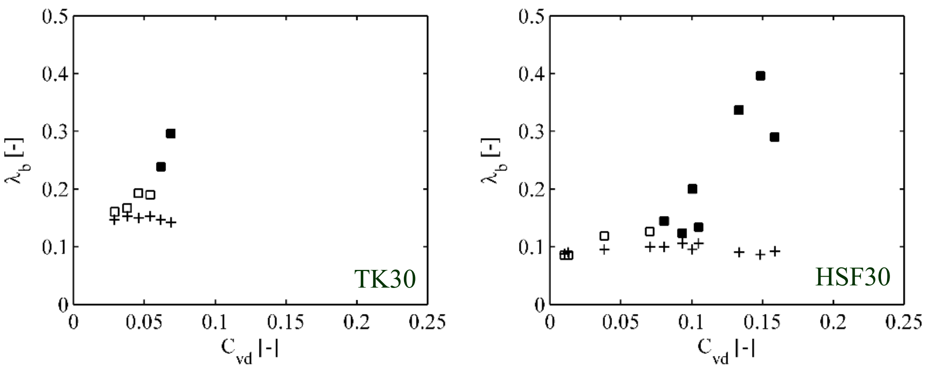

Figure 2 Bed friction coefficient versus delivered concentration of sediment at constant mixture flow rate ($Q_m \approx 7.4$ l/s) in upper plane bed regime. Legend: blank squares –measurements in lower sub-regime (l-UPB); black squares – measurements in upper sub-regime (u-UPB), plus marks – predictions using Equation 3 for appropriate value of Δ ($\Delta = 7.0$ for TK30, $\Delta = 4.0$ for HSF30).

4. CONCLUSIONS

In case of immobile grains (the lower plane bed regime), the grain size is the only relevant parameter through which sediment affects the bed friction coefficient. In the logarithmic law for the top of the lower plane bed (not eroded by the flow), the Nikuradse's equivalent roughness of the bed can be estimated as twice the size of a sediment grain.

Mobile grains at the top of bed (the upper plane bed regime) affect the bed friction coefficient not only through their size but also through their density. In the lower sub-regime (where the grain transport layer is a smaller part of the total flow depth and hence the log profile of velocity spans over a major part of the depth), the log law is still appropriate to describe the bed friction coefficient. The Nikuradse's bed roughness can be expressed as a multiple of the sediment-grain size but it is bigger and, contrary to the lower plane bed, the multiple seems to be sensitive to the sediment-grain density (it increases with the sediment density). In the upper sub-regime, the transport layer dominates the flow depth and the thickness of the water layer with the log distribution of velocity is small. Hence, the log law of the wall is not appropriate for the top of the bed and the Nikuradse's equivalent roughness is irrelevant. Instead, a different law must be used and it should take into account the observed trend of a lower value of the bed friction coefficient for the lightweight sediment than for the natural sediment at the same delivered concentration of bed load in the channel.

In general, the use of lightweight sediment in laboratory experiments should pose no problem in model scaling of the roughness of the lower plane bed but different densities of the model material and the prototype material need to be taken into account in scaling of the friction coefficient for the upper plane bed as the lightweight material may considerably underestimate the bed friction coefficient at a given delivered concentration of sediment.

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